Importance Sampling of Area Lights in Participating Media

Christopher Kulla Marcos Fajardo

Outline

- Previous Work
- Single Scattering Equation
- Importance Sampling for Point Lights
- Importance Sampling for Area Lights
- Results

Previous Work - Unbiased Methods

- “Ray tracing volume densities” [Kajiya and Von Herzen, 1984]
- “Unbiased Global Illumination with Participating Media” [Raab et al, 2006]

Previous Work - Analytical Methods

- “A Practical Analytic Single Scattering Model for Real Time Rendering” [Sun et al, 2005]
- “An Analytical Solution to Single Scattering in Homogeneous Participating Media” [Pegoraro et al, 2009]
- “A Mathematical Framework for Efficient Closed-Form Single Scattering” [Pegoraro et al, 2011]

Previous Work - Realtime Shadowing

- “Interactive Volumetric Shadows in Participating Media with Single-Scattering” [Wyman et al, 2008]
- “Epipolar Sampling for Shadows and Crepuscular Rays in Participating Media with Single Scattering” [Engelhardt, 2010]
- “Real-Time Volumetric Shadows using 1D Min-Max Mipmaps” [Chen et al, 2011]
- “Voxelized Shadow Volumes” [Wyman et al, 2011]

Previous Work - Offline Methods

- "Radiance Caching for Participating Media" [Jarosz et al, 2008]
- "A Comprehensive Theory of Volumetric Radiance Estimation using Photon Points and Beams" [Jarosz et al, 2011]

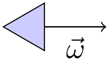
Our Contributions

We will focus on importance sampling of single scattering (direct lighting):

- Unbiased
- No memory requirements
- Simple implementation
- Easy integration into any Monte Carlo based renderer

Single Scattering Equation for Point Light

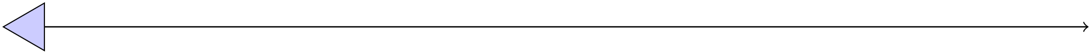
We want to evaluate radiance L through a pixel



$$L(x, \vec{\omega}) =$$

Single Scattering Equation for Point Light

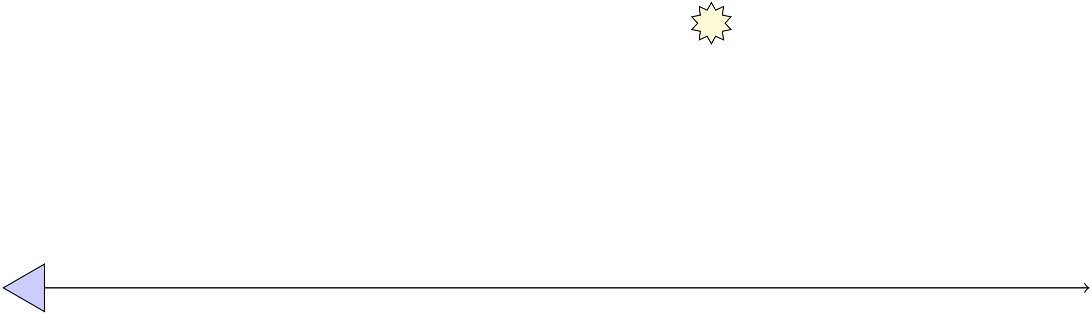
Trace a ray into the homogeneous medium



$$L(x, \vec{\omega}) =$$

Single Scattering Equation for Point Light

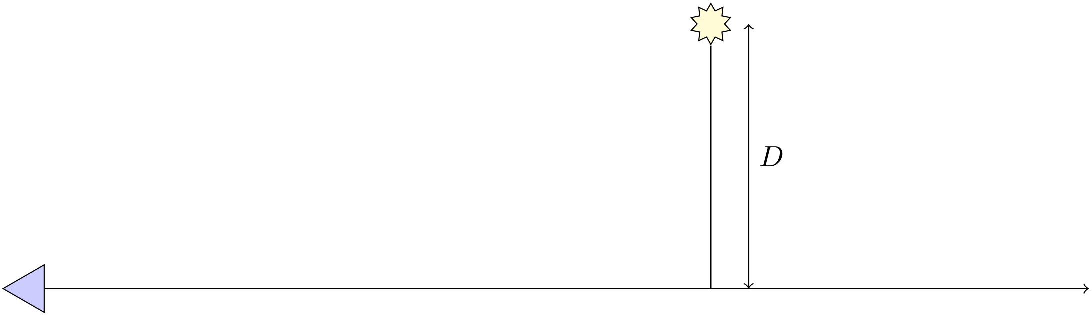
Point Light with power Φ



$$L(x,\vec{\omega}) = \qquad \frac{\Phi}{}$$

Single Scattering Equation for Point Light

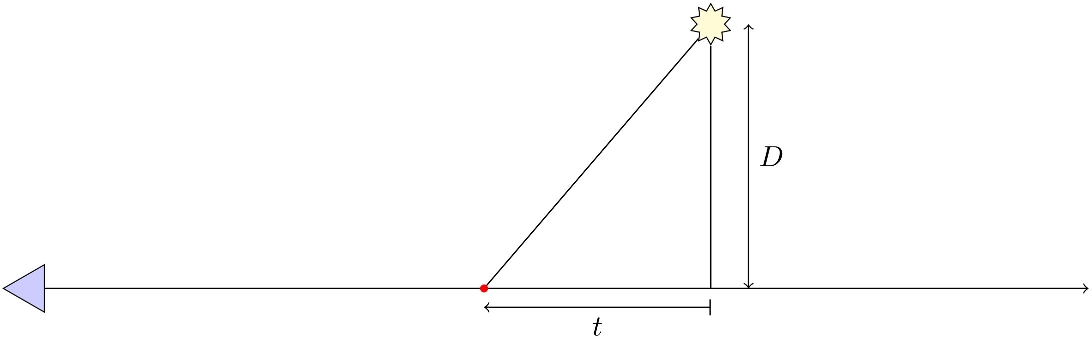
Point Light is a distance D from the ray



$$L(x, \vec{\omega}) = \qquad \frac{\Phi}{D^2}$$

Single Scattering Equation for Point Light

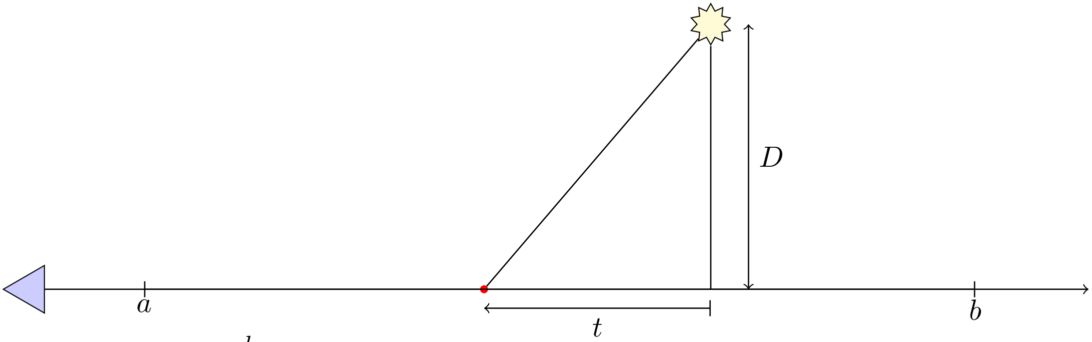
Contributes to point t along the ray (measured from projection point)



$$L(x, \vec{\omega}) = \qquad \frac{\Phi}{D^2 + t^2}$$

Single Scattering Equation for Point Light

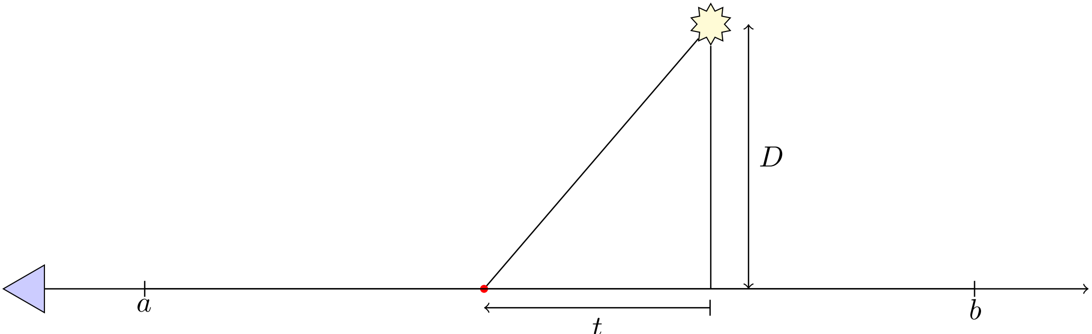
Integrate contribution between a and b



$$L(x,\vec{\omega}) = \int_a^b \frac{\Phi}{D^2+t^2}\,dt$$

Single Scattering Equation for Point Light

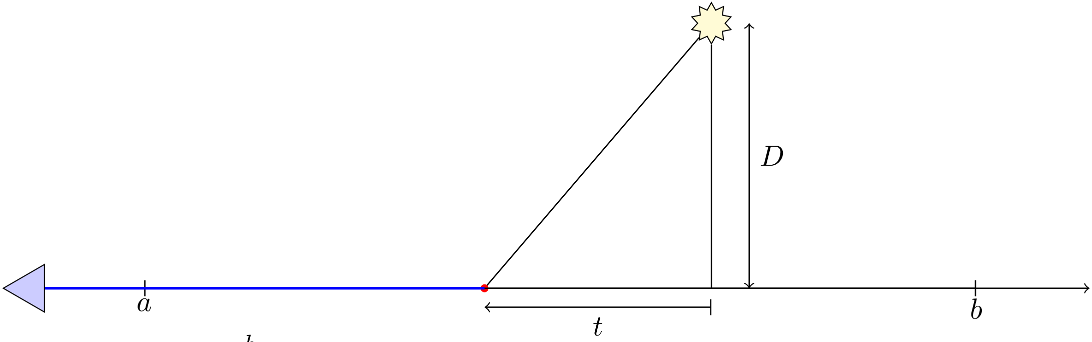
Account for scattering coefficient σ_s



$$L(x, \vec{\omega}) = \sigma_s \int_a^b \qquad \frac{\Phi}{D^2+t^2}\, dt$$

Single Scattering Equation for Point Light

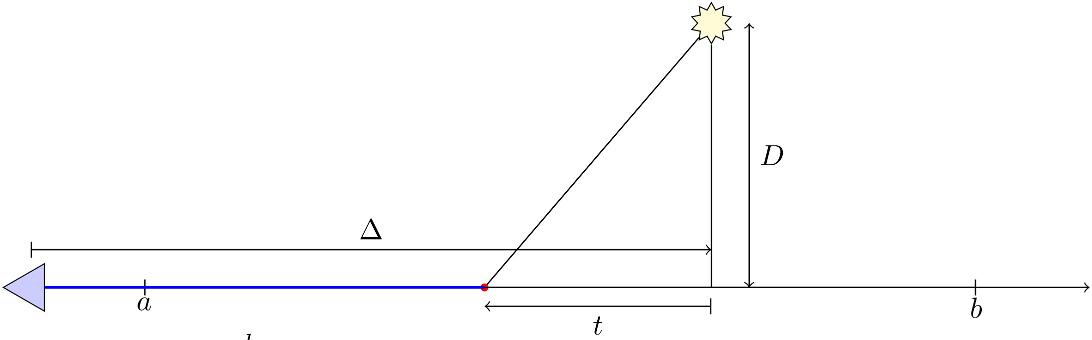
Account for extinction (σ_t) up to sample point



$$L(x, \vec{\omega}) = \sigma_s \int_a^b e^{-\sigma_t \left(t \qquad\qquad\qquad\qquad \right)} \frac{\Phi}{D^2 + t^2}\, dt$$

Single Scattering Equation for Point Light

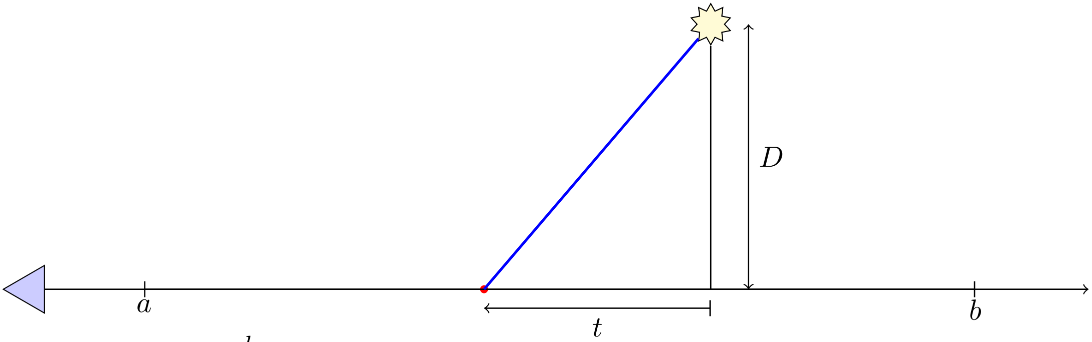
To account for change of variables, we add the signed distance Δ from ray origin



$$L(x, \vec{\omega}) = \sigma_s \int_a^b e^{-\sigma_t\left(t+\Delta \qquad\qquad\right)} \frac{\Phi}{D^2+t^2}\, dt$$

Single Scattering Equation for Point Light

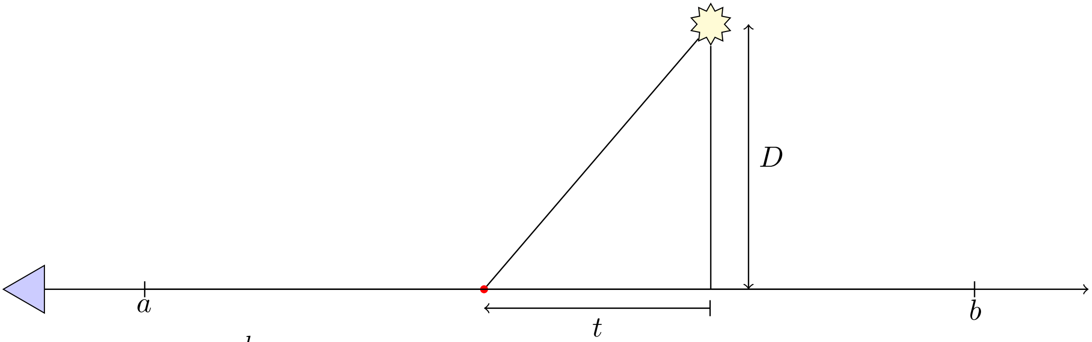
Finally, add extinction towards the light



$$L(x, \vec{\omega}) = \sigma_s \int_a^b e^{-\sigma_t\left(t+\Delta+\sqrt{D^2+t^2}\right)} \frac{\Phi}{D^2+t^2}\, dt$$

Single Scattering Equation for Point Light

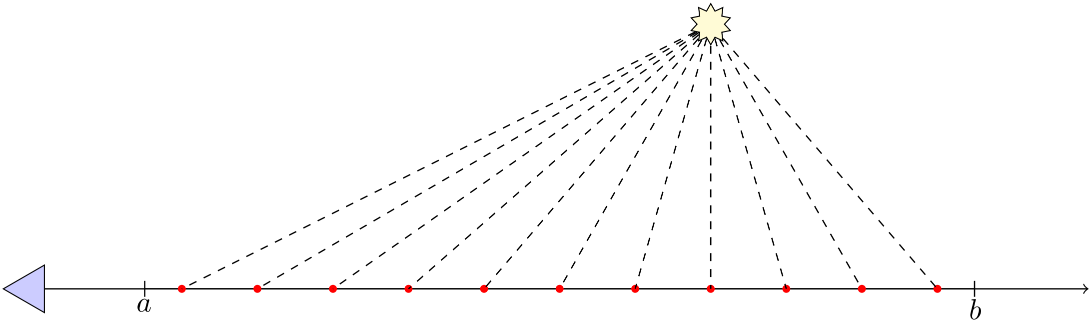
Omit phase function from equation to simplify the notation



$$L(x,\vec{\omega}) = \sigma_s \int_a^b e^{-\sigma_t\left(t+\Delta+\sqrt{D^2+t^2}\right)} \frac{\Phi}{D^2+t^2}\, dt$$

Single Scattering Equation for Point Light

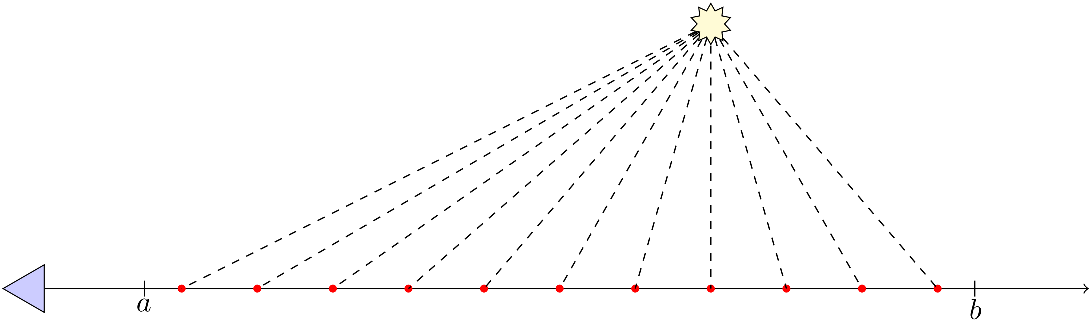
To evaluate the integral we take n samples along the line



$$L(x,\vec{\omega}) = \frac{\sigma_s}{n}\sum_{i=1}^{n}\left(e^{-\sigma_t(t_i+\Delta+\sqrt{D^2+t_i^2})}\frac{\Phi}{D^2+t_i^2}\right)/\mathsf{pdf}(t_i)$$

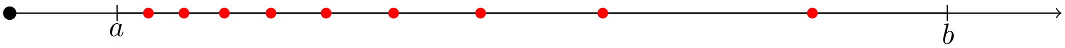
Single Scattering Equation for Point Light

How should these samples be distributed?



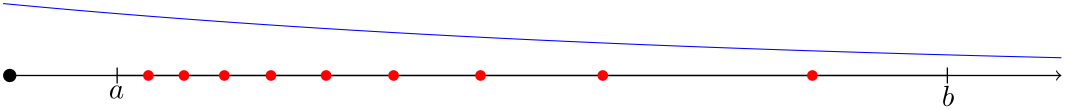
$$L(x,\vec{\omega}) = \frac{\sigma_s}{n}\sum_{i=1}^{n}\left(e^{-\sigma_t\left(t_i+\Delta+\sqrt{D^2+t_i^2}\right)}\frac{\Phi}{D^2+t_i^2}\right)\Big/\mathsf{pdf}(t_i)$$

Density distribution



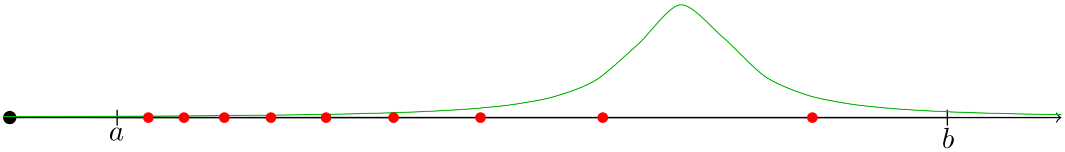
- Place samples proportionally to attenuation?

Density distribution



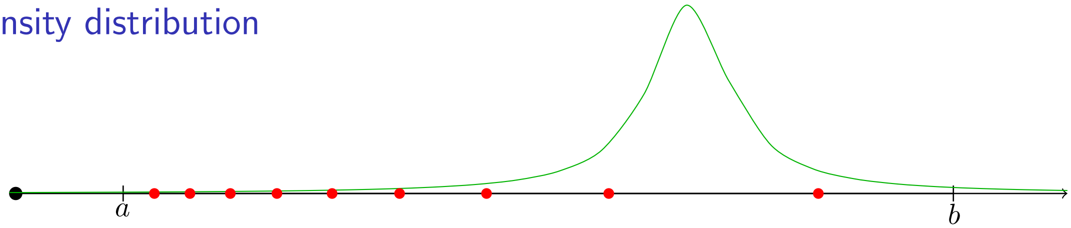
- Place samples proportionally to attenuation?
- Attenuation is bounded by 1 and varies smoothly

Density distribution



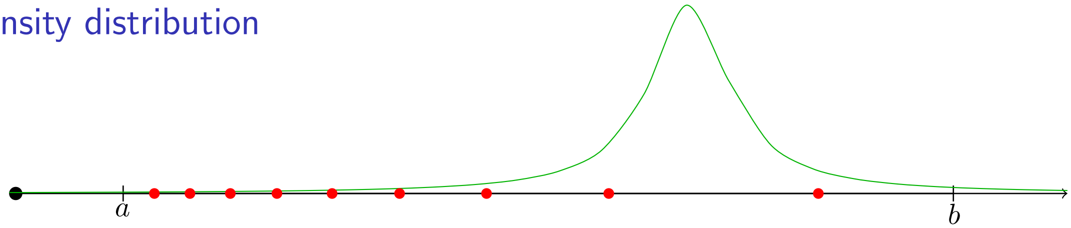
- Place samples proportionally to attenuation?
- Attenuation is bounded by 1 and varies smoothly
- Lighting term varies as $1/r^2$

Density distribution

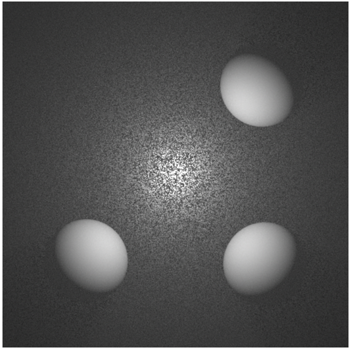


- Place samples proportionally to attenuation?
- Attenuation is bounded by 1 and varies smoothly
- Lighting term varies as $1/r^2$
- Dominates as we get closer to the light

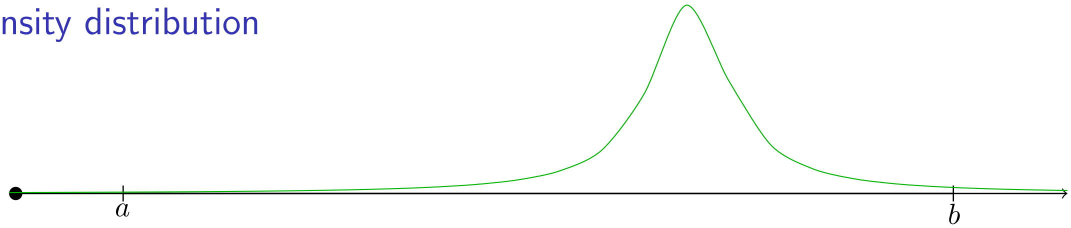
Density distribution



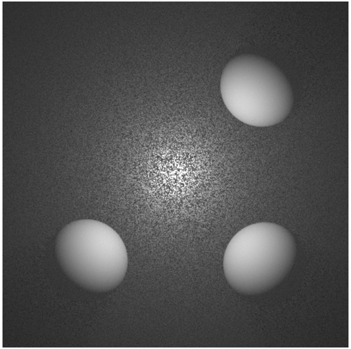
- Place samples proportionally to attenuation?
- Attenuation is bounded by 1 and varies smoothly
- Lighting term varies as $1/r^2$
- Dominates as we get closer to the light



Density distribution



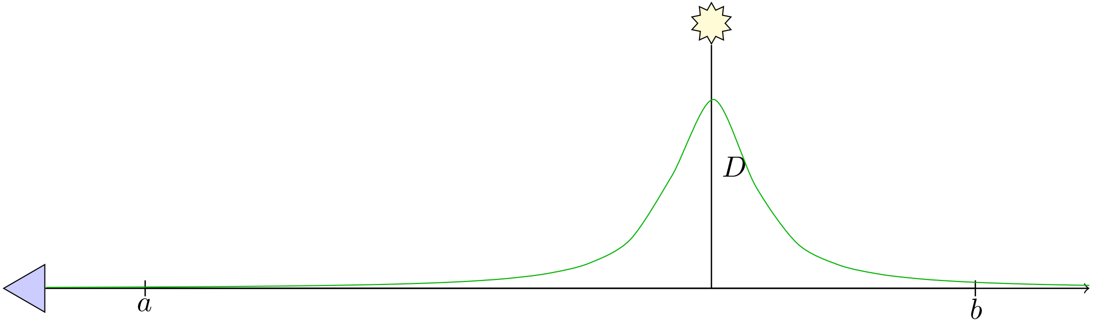
- Place samples proportionally to attenuation?
- Attenuation is bounded by 1 and varies smoothly
- Lighting term varies as $1/r^2$
- Dominates as we get closer to the light
- Can we design a pdf proportional to lighting term?



Improving the distribution

Goal is to get a pdf proportional to lighting term:

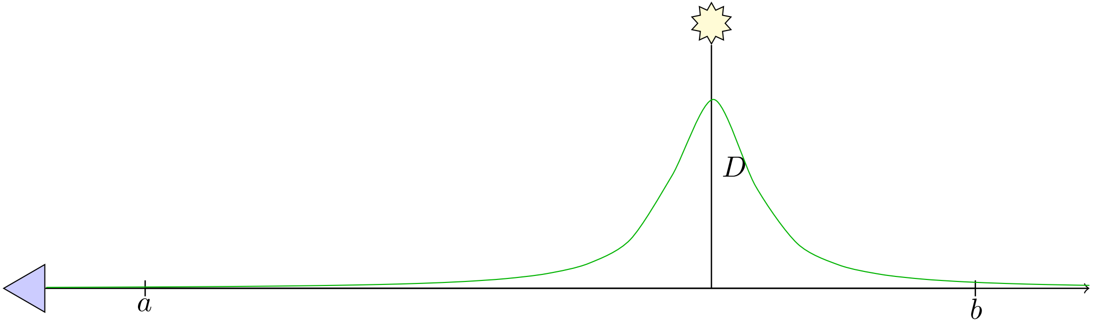
$\mathsf{pdf}(t) \propto \frac{1}{D^2+t^2}$



Improving the distribution

Integrate pdf to obtain cdf:

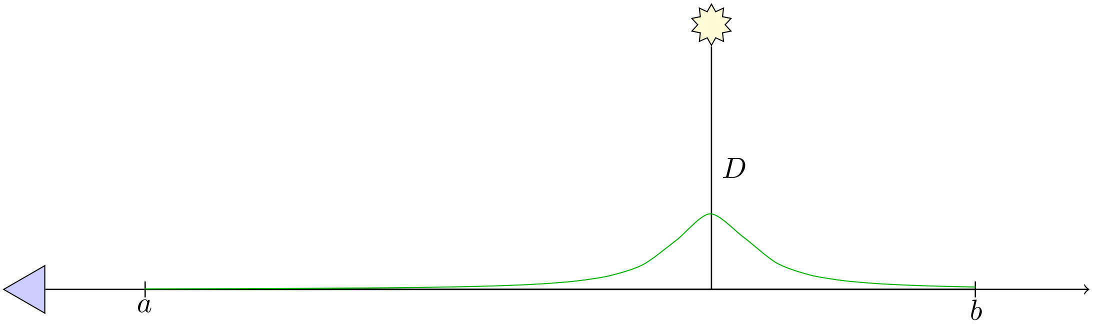
$$\mathsf{cdf}(t) = \int \frac{1}{D^2+t^2}\,dt = \frac{1}{D}\tan^{-1}\frac{t}{D}$$



Improving the distribution

Use cdf to normalize over $[a, b]$:

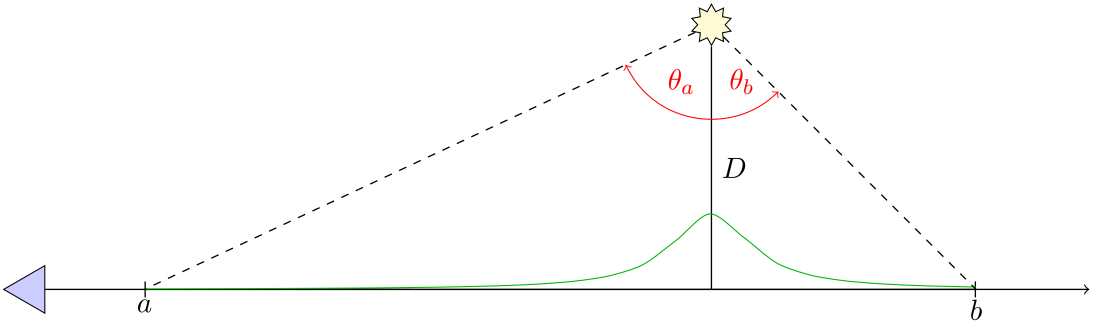
$$\mathsf{pdf}(t) = \frac{D}{(\tan^{-1}\frac{b}{D} - \tan^{-1}\frac{a}{D})(D^2 + t^2)}$$



Improving the distribution

Use cdf to normalize over $[a, b]$:

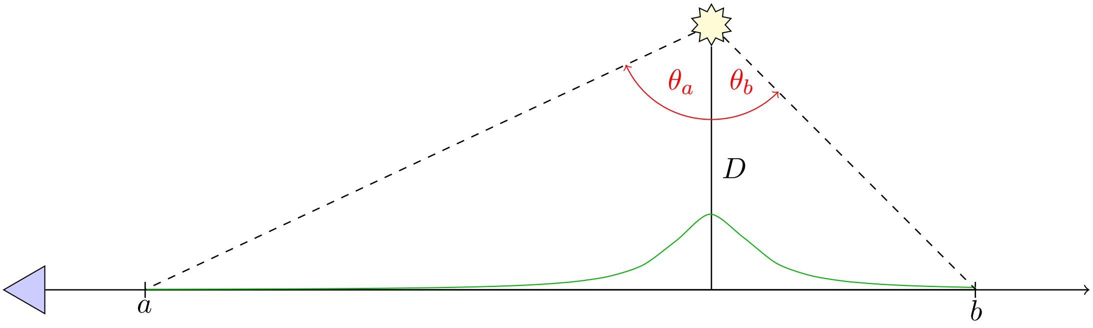
$$\mathsf{pdf}(t) = \frac{D}{(\theta_b - \theta_a)(D^2 + t^2)}$$



Improving the distribution

Invert cdf to obtain distribution for $\xi_i \in [0, 1)$:

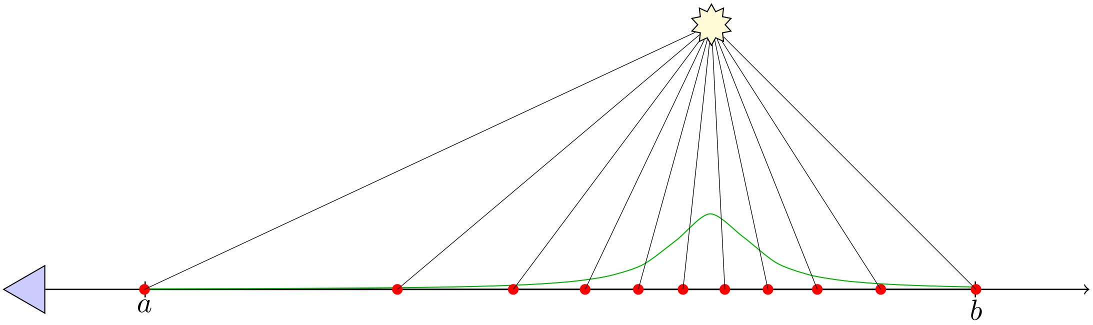
$$t_i = D \tan\left(\left(1 - \xi_i\right)\theta_a + \xi_i \theta_b\right)$$



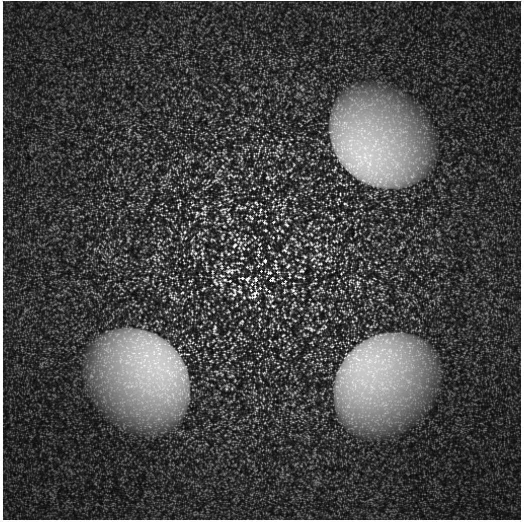
Improving the distribution

Sample distribution is *equi-angular*

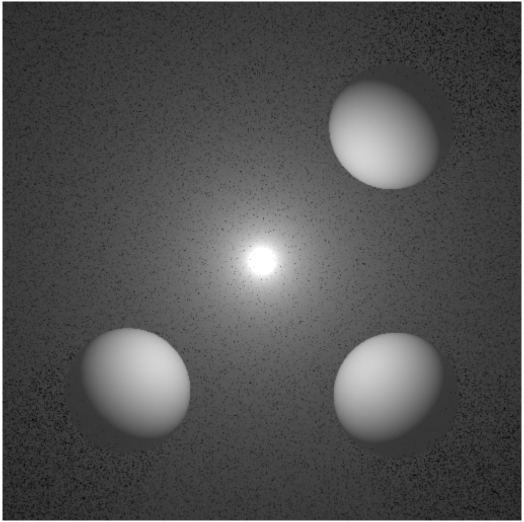
$$t_i = D\tan\left(\left(1-\xi_i\right)\theta_a + \xi_i\theta_b\right)$$



Results with 1 sample/pixel

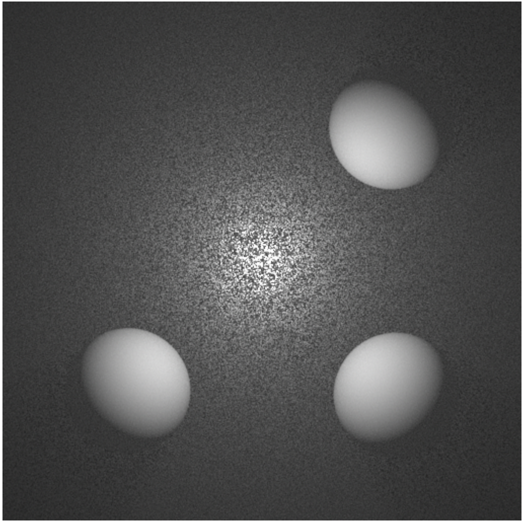


Density sampling

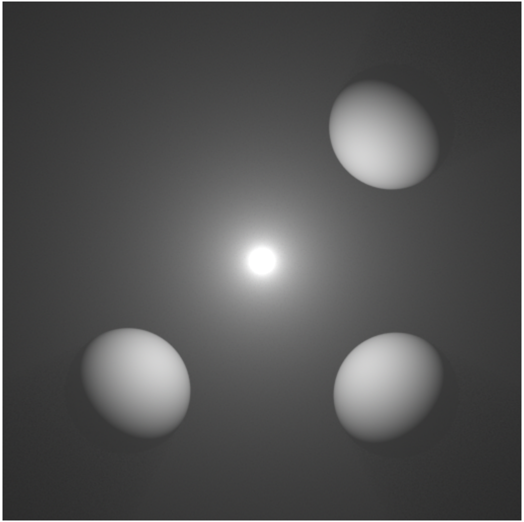


Our method

Results with 16 samples/pixel

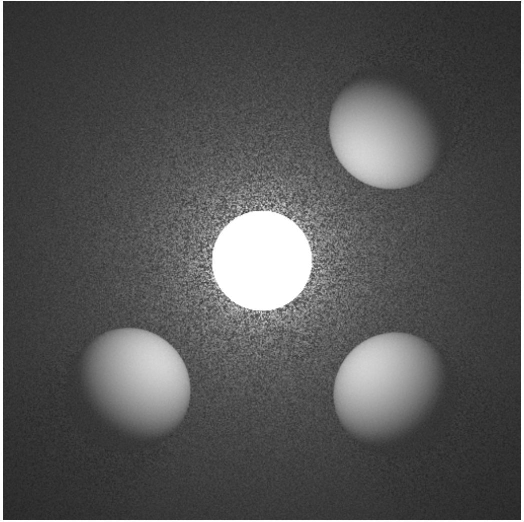


Density sampling

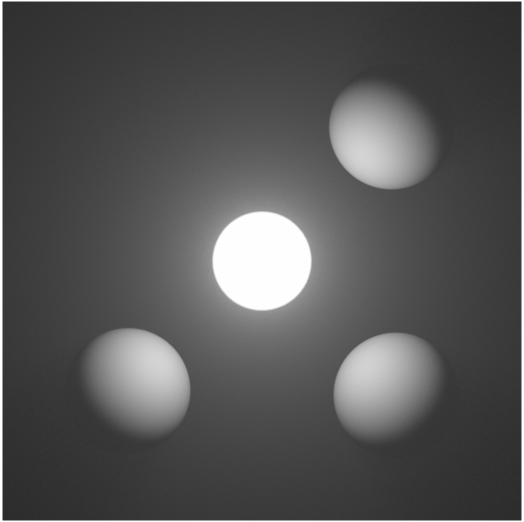


Our method

Sphere lights can use same equations!

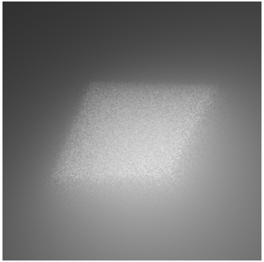


Density sampling

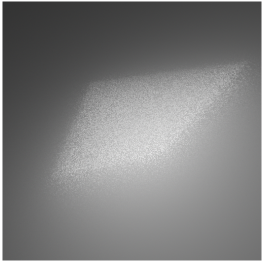


Our method

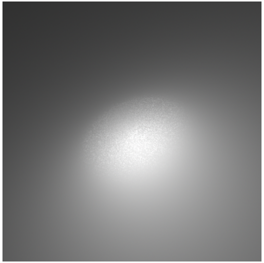
What about general area lights?



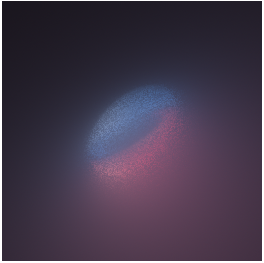
Rectangular



Generalized Quad



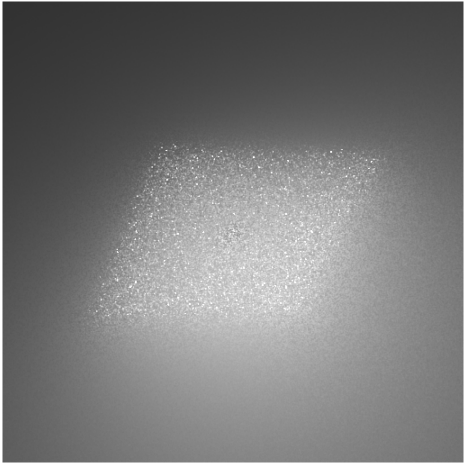
Disc



Textured

Single Scattering from Area Lights (first attempt)

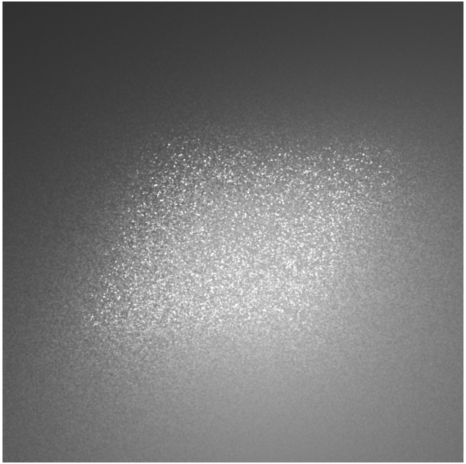
- Apply equi-angular sampling from center



Centered Equi-angular Sampling
256 samples / pixel

Single Scattering from Area Lights (first attempt)

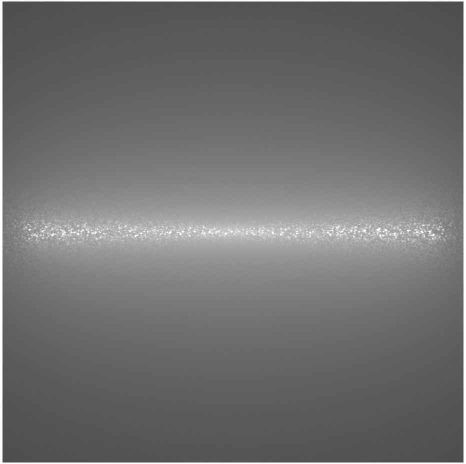
- Apply equi-angular sampling from center
- Better results than density sampling



Density Sampling
256 samples / pixel

Single Scattering from Area Lights (first attempt)

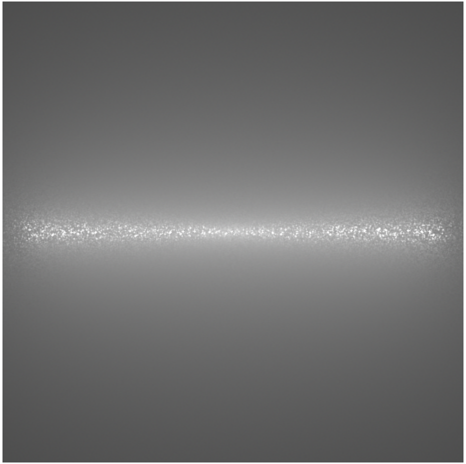
- Apply equi-angular sampling from center
- Better results than density sampling
- But error increases away from the center
- Can be arbitrarily bad for wide lights



Centered Equi-angular Sampling
256 samples / pixel

Single Scattering from Area Lights (second attempt)

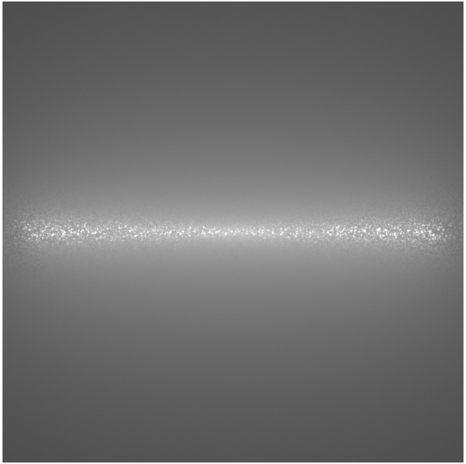
- Exchange integral over light area with line integral



Centered Equi-angular Sampling
256 samples / pixel

Single Scattering from Area Lights (second attempt)

- Exchange integral over light area with line integral
- Choose the light sample point *first*



Centered Equi-angular Sampling
256 samples / pixel

Single Scattering from Area Lights (second attempt)

- Exchange integral over light area with line integral
- Choose the light sample point *first*
- *Then* apply equi-angular sampling

Varying Equi-angular Sampling
256 samples / pixel

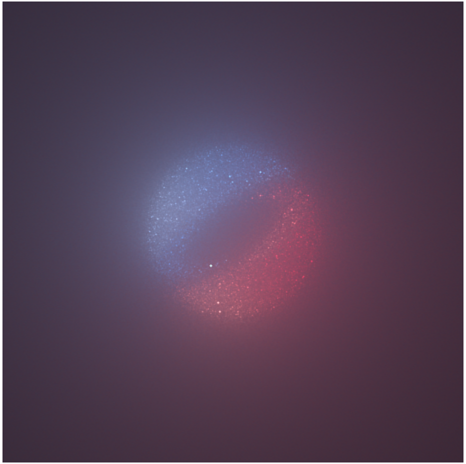
Single Scattering from Area Lights (second attempt)

- Exchange integral over light area with line integral
- Choose the light sample point *first*
- *Then* apply equi-angular sampling
- Error is now more uniformly distributed

Varying Equi-angular Sampling
256 samples / pixel

Single Scattering from Area Lights

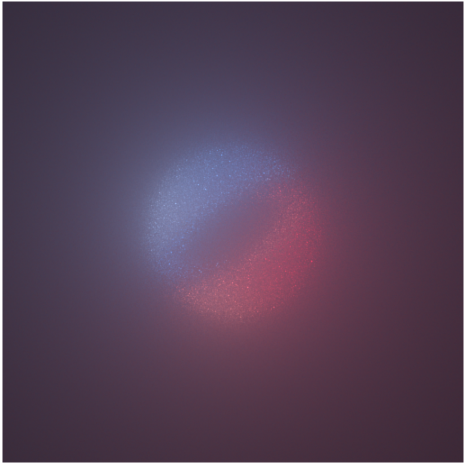
- Some high variance speckles remain
- $1/(D^2 + t^2)$ has a singularity in D as well



Varying Equi-Angular Sampling

Single Scattering from Area Lights

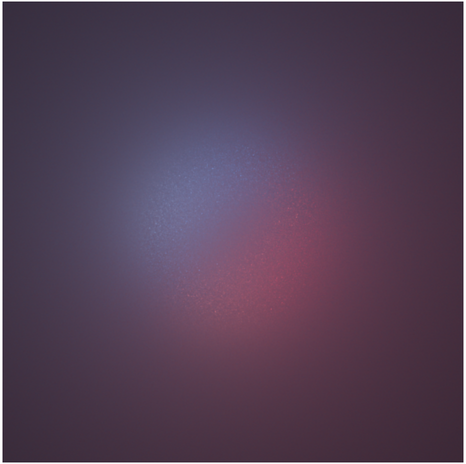
- Some high variance speckles remain
- $1/(D^2 + t^2)$ has a singularity in D as well
- Can mask these by clamping



Clamp < 0.05

Single Scattering from Area Lights

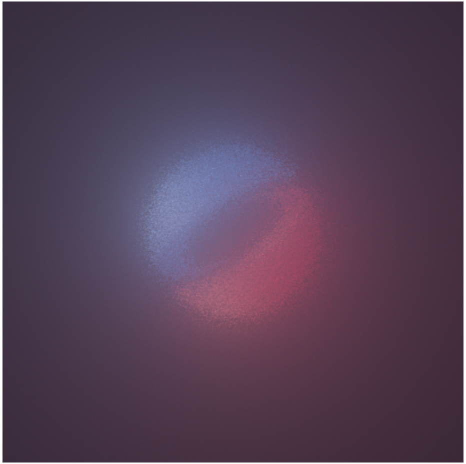
- Some high variance speckles remain
- $1/(D^2 + t^2)$ has a singularity in D as well
- Can mask these by clamping (biased!)



Clamp < 0.50 (too high!)

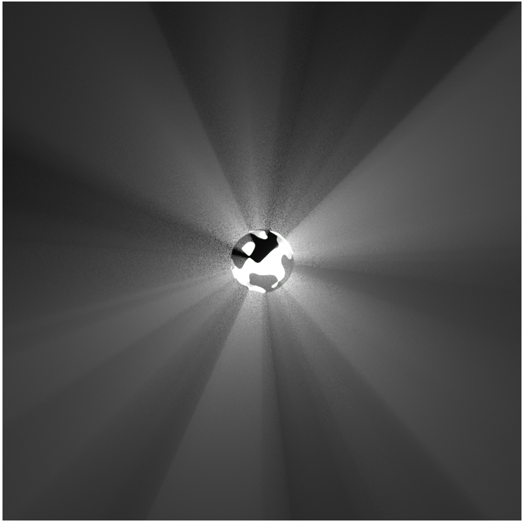
Single Scattering from Area Lights

- Some high variance speckles remain
- $1/(D^2 + t^2)$ has a singularity in D as well
- Can mask these by clamping (biased!)
- Or by applying MIS

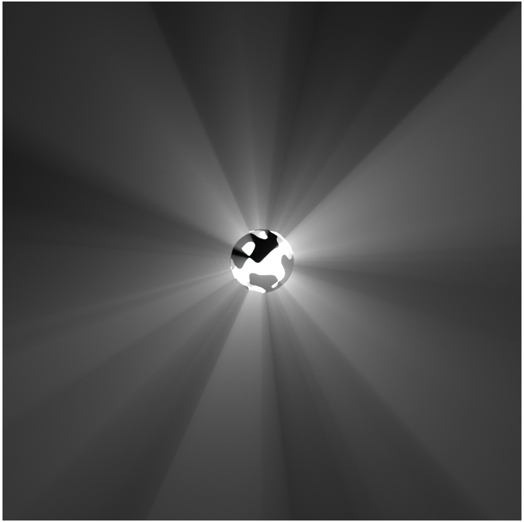


MIS with phase function sampling

Examples (64 samples / pixel)

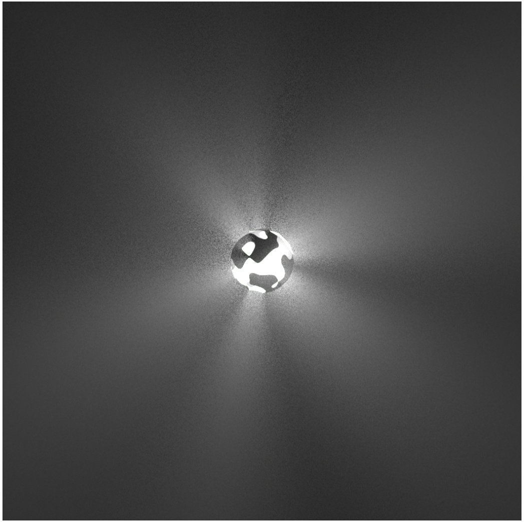


Density sampling

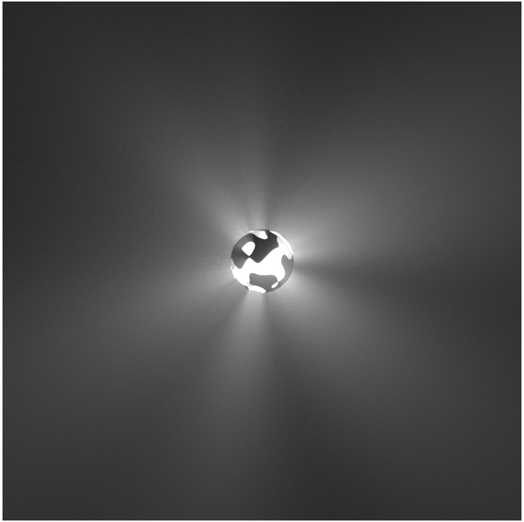


Our method

Examples (64 samples / pixel)

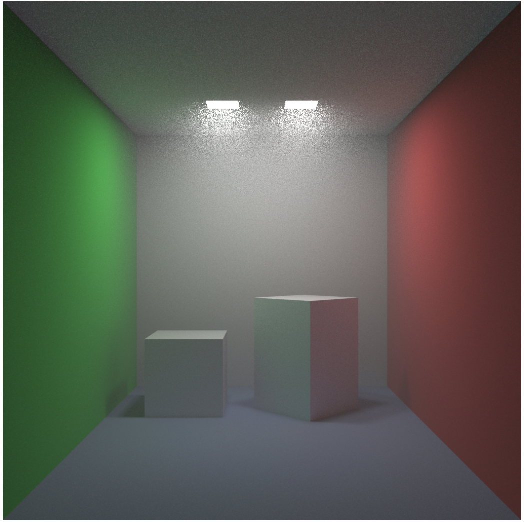


Density sampling

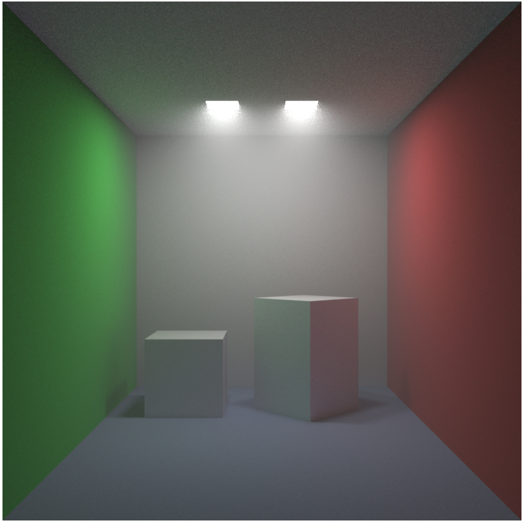


Our method

Examples (16 samples / pixel)

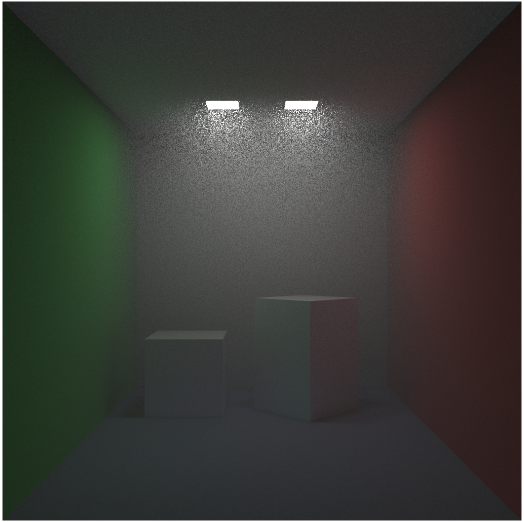


Density sampling

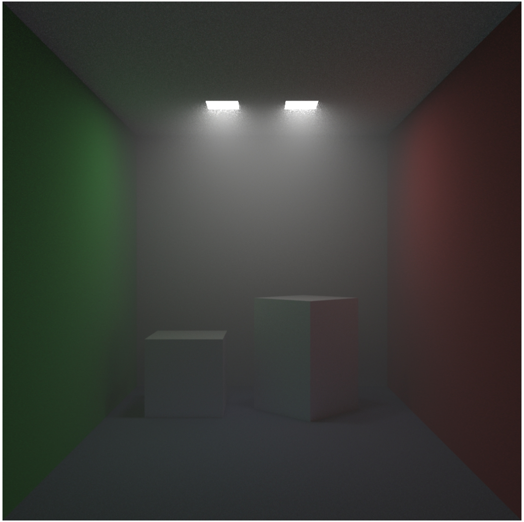


Our method

Examples (16 samples / pixel)

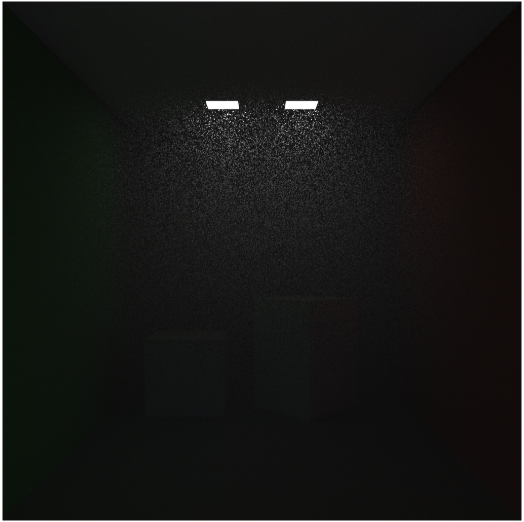


Density sampling

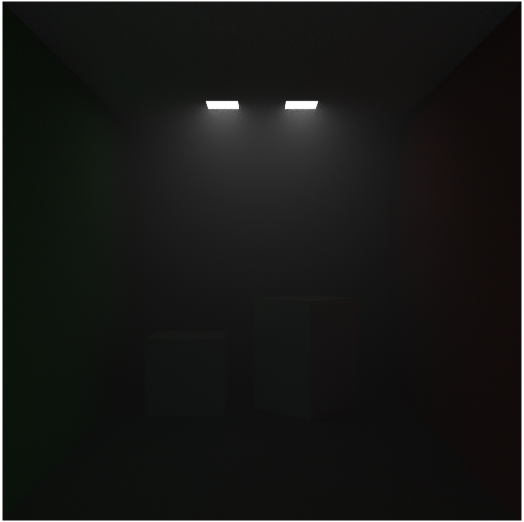


Our method

Examples (16 samples / pixel)



Density sampling



Our method

Summary

- Equi-angular importance sampling for point and spherical lights
- Simple extension to arbitrary area lights
- Very simple implementation
- No restrictions on motion blur or visibility

Future Work

- Region close to light surface remains noisy
- Explore analytical solutions for rectangles and discs
- Incorporate phase function into estimate
- Apply to bidirectional path-tracing (camera behaves like a point light)

Thanks for listening!